SET :- A set is a Coll^n of objects.

for eg:- 1) set of students in sec D.

2) set of Girls in sec D.

3) Set of Boys in sec D.

4) set of teachers in School.

Set of number from 1 to 10. i.e.

$G = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Set of vowels in English letter

$H = \{a, e, i, o, u\}$.

~~Set of~~ $I = \{1, 2, 3, 4\}$.

$\mathbb{N} \rightarrow$ set of natural no.

$\mathbb{N} = \{1, 2, 3, 4, 5, 6, \ldots\}$

$\mathbb{Q} \rightarrow$ set of rational no. ie. p/q form where $q \neq 0$

$\mathbb{Q}^c \rightarrow$ set of irrational no. for eg. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \ldots$

$\mathbb{R} \rightarrow$ set of real no.'s.

$\mathbb{C} \rightarrow$ set of complex no.

for eg. $a+ib$ form, where $a, b \in \mathbb{R}$.

$\mathbb{Z} \rightarrow$ set of integers.

$\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

Group

Let G be a non-empty set and "$\circ$" is any operation. Then $(G, \circ)$ is said to be group if satisfies following Properties :-

(1) Closure Property :- $\forall$ $a \in G$, $\forall$ $b \in G$ such that (s.t.) — for all
$a \circ b \in G$. [here $\in \rightarrow$ belong to]

(2) Associative Law :- $a \circ (b \circ c) = (a \circ b) \circ c$ $\forall$ $a, b, c \in G$.

(3) Eixtence of identity :- $\forall$ $a \in G$ there exist Unique element $e \in G$ such that (s.t.)

$$a \circ e = e \circ a = a.$$

(4) Existence of Inverse :- For each $a \in G$, there exit ($\exists$) $a^{-1} \in G$ such that $a \circ a^{-1} = a^{-1} \circ a = e$.

Q 1.) Show that $(\mathbb{Z}, +)$ is group.

Proof $\mathbb{Z} = \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots\}$

(i) let $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$ such that $a+b$ is also an integer.

$\Rightarrow a+b \in \mathbb{Z}$, $\forall$ $a \in \mathbb{Z}, b \in \mathbb{Z}$.

Closure Property Hold.

(ii) $a + (b+c) = (a+b)+c$ $\forall$ $a, b, c \in \mathbb{Z}$.

$\Rightarrow$ Associative Property hold.

(iii) $0 \in \mathbb{Z}$ s.t. $a+0 = 0+a = a, \forall a \in \mathbb{Z}$.

i.e. there ~~exist~~ exist identity in $\mathbb{Z}$.

(iv) let $a \in \mathbb{Z}$ then $-a \in \mathbb{Z}$ s.t. (such that)

$$a + (-a) = (-a) + a = 0 .$$

$\Rightarrow$ then $(\mathbb{Z}, +)$ is group.

Q2. To show $(\mathbb{Q}, +)$, $(\mathbb{R}, +)$, $(\mathbb{C}, +)$ are groups with identity 0 (zero) and inverse $-a$.

Q.N. Show that $(\mathbb{Z}^*, \cdot)$ is group.

Proof $\mathbb{Z}^* = \{1, -1\}$

	1	-1
1		
-1		

$\Rightarrow$

	1	-1
1	1	-1
-1	-1	1

(i) Let $\forall a \in \mathbb{Z}^*, \forall b \in \mathbb{Z}^*$ s.t. $a \cdot b \in \mathbb{Z}^*$

Closure Property hold.

(2) $a \cdot (b \cdot c) = (a \cdot b) \cdot c \ \forall a, b, c \in \mathbb{Z}^*$. (Associative Property hold)

(3) $1 \in \mathbb{Z}$ s.t. $a \cdot 1 = a \ \forall a \in \mathbb{Z}^*$.

there exist identity $1 \in \mathbb{Z}^*$.

(4) Inverse of each element of $\mathbb{Z}^*$.

$$1^{-1} = 1$$

$$(-1)^{-1} = 1$$

then $(\mathbb{Z}^*, \cdot)$ is group.

Q Show that $\mathbb{Z}_n$ is group w.r.to. addition (+) under modulo n.

Proof :- ~~Let $a \in \mathbb{Z}_n$ then~~

$$\mathbb{Z}_n = \{0, 1, 2, 3, \dots (n-1)\}.$$

(1) let $a \in \mathbb{Z}_n$ then $0 \le a \le n-1$ and $b \in \mathbb{Z}_n$
$0 \le b \le n-1$
s.t. $0 \le a+b \le n-1$ w.r.to modulo n.
then $a+b \in \mathbb{Z}_n$. (Closure Property hold)

(ii) $a+(b+c) = (a+b)+c \quad \forall a, b, c \in \mathbb{Z}_n$.
$\Rightarrow$ Associative Property hold.

(III) since $n \ge 1$, then $0 \in \mathbb{Z}_n$ s.t.
$a+0 = 0+a = a$
$\forall a \in \mathbb{Z}_n$.
$\exists$ (there exist) identity $0 \in \mathbb{Z}_n$.

(IV) let $a \in \mathbb{Z}_n$ and $n-a \in \mathbb{Z}_n$ w.r.to n
s.t. $a+(n-a) = (n-a)+a = 0$ 'w.r.to modulo n.'
$\exists$ inverse of $a \in \mathbb{Z}_n$ is $n-a$.

Hence $(\mathbb{Z}_n, +)$ is group.

Q Find universe of each element of $\mathbb{Z}_6$.

soln:- $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$

here 0 is identity of $\mathbb{Z}_6$.

universe of 1 is $(6-1) = 5$

i.e. $1^{-1} = 5$

inverse of 2 is $(6-2) = 4$

i.e. $2^{-1} = 4$

inverse of 3 is $(6-3) = 3$

i.e. $3^{-1} = 3$

inverse of 4 is $(6-4) = 2$

i.e. $4^{-1} = 2$

inverse of 5 is $(6-5) = 1$

i.e. $5^{-1} = 1$

Q Find inver of each element of $\mathbb{Z}_7$.

Q find universe of each element of $\mathbb{Z}_{10}$

Q find inverse of each element of $\mathbb{Z}_{11}$.